

Measurability of Nuclear Electric Dipole Moments*

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The possibility of measuring a very small nuclear electric dipole moment is explored by calculating the interaction of this moment with an external electric field. It is shown that for a quantum system of point, charged, electric dipoles in an external electrostatic potential of arbitrary form, there is complete shielding; i.e., there is no term in the interaction energy that is of first order in the electric dipole moments, regardless of the magnitude of the external potential. This is true even if the particles are of finite size, provided that the charge and dipole moment of each have the same spatial distribution. Relativistic and second-order effects are uninterestingly small. There is, however, a first-order interaction if the charge and moment distributions are different, and also for a point electric dipole if it also carries a magnetic dipole moment. Explicit calculations of both effects are given for hydrogen and helium atoms. It is found that the effective electric field at a He^3 nucleus arising from the magnetic dipole effect is about a hundred times that arising from the finite size effect, and is roughly 10^{-7} times the external electric field.

I. INTRODUCTION

SHORTLY after the suggestion by Lee and Yang¹ that parity (P) is not conserved in weak interactions, Landau² pointed out that invariance under the combined operation CP of charge conjugation (C) and parity is needed to rule out the existence of static electric dipole moments of elementary particles. While there is ample evidence that the weak interactions are not invariant under C and P separately, they may be invariant under CP .³ The CPT theorem would then imply invariance under time reversal (T). However, observations on the correlation between the neutron spin vector and the proton and electron momentum vectors in the decay of polarized neutrons leave open the possibility of an appreciable breakdown of T invariance,⁴ and other kinds of experiments do not appear to restrict this possibility significantly.⁵ Thus, it is worthwhile to consider attempting the measurement of a nuclear electric dipole moment, or indeed of any "odd" nuclear moment (magnetic monopole or quadrupole, electric octupole, etc.). The magnetic monopole has been the subject of recent searches,⁶ and we shall not consider it further.

Measurement of higher "odd" moments is subject to the following general difficulty. Nuclei are expected to, and as far as is known do, possess those "even" multipole moments that are allowed by angular momentum considerations. Within 1 or 2 orders of magnitude, these

moments are determined by the nuclear size. Such an electric quadrupole moment, for example, interacts with an environmental electric field of appropriate symmetry. Any magnetic quadrupole moment that might be present will interact in exactly the same way with an environmental magnetic field of the same symmetry. Thus, in order to detect the exceedingly small magnetic quadrupole moment that would be expected on the basis of a possible breakdown of T invariance in weak interactions, the environmental electric field must be made exceedingly small in comparison with the magnetic field of the same symmetry, and both this electric field and the electric quadrupole moment must be known with great accuracy. Reduction of the unwanted field by the requisite amount, and sufficiently precise knowledge of the unwanted field and moment, appear to be impossibly difficult of attainment when the environment consists of condensed matter, as it must if there is to be a measurable interaction in the case of quadrupole or higher moments.

In this respect, the measurement of an electric dipole moment in the presence of a much larger magnetic dipole moment is relatively favorable, since the environmental fields can be supplied by laboratory equipment and need not be produced by nearby atoms. In this way, Smith, Purcell, and Ramsey⁷ attempted to measure the change in the precession frequency of neutrons in a weak uniform magnetic field when a strong uniform electric field was superposed parallel to the magnetic field. They found that if the neutron electric dipole moment is written as eD , where e is the electronic charge, then $|D| < 5 \times 10^{-20}$ cm.

This idea has been carried to its limit by Fairbank,⁸ who has proposed that a precession experiment be performed in zero magnetic field and a strong electric field, in which case any observed precession must arise from an electric dipole moment. It now appears possible in

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¹ T. D. Lee and C. N. Yang, *Phys. Rev.* **104**, 254 (1956).

² L. Landau, *Nucl. Phys.* **3**, 127 (1957).

³ For a review of the experiments and their interpretation, see E. J. Konopinski, *Ann. Rev. Nucl. Sci.* **9**, 99 (1959).

⁴ M. A. Clark and J. M. Robson, *Can. J. Phys.* **38**, 693 (1960); M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, *Phys. Rev.* **120**, 1829 (1960).

⁵ For a discussion of this point, see R. G. Sachs, *Phys. Rev.* **129**, 2280 (1963).

⁶ E. M. Purcell, G. B. Collins, T. Fujii, J. Hornbostel, and F. Turkot, *Phys. Rev.* **129**, 2326 (1963); E. Amaldi, G. Baroni, A. Manfredini, H. Bradner, L. Hoffmann, and G. Vanderhaeghe, *Nuovo Cimento* **28**, 773 (1963).

⁷ J. H. Smith, E. M. Purcell, and N. F. Ramsey, *Phys. Rev.* **108**, 120 (1957).

⁸ W. M. Fairbank (private communication).

principle to construct regions of space in which the magnetic field is exactly zero by making use of the phenomenon of flux quantization.^{8,9} Great sensitivity can then be achieved by looking for precession through a very small angle in a time of the order of hours or days, provided that the precessing objects can be held without other disturbance in the magnetic-field-free region for this length of time. This, of course, rules out the use of neutrons, and suggests that the experiment be performed on nuclei. It appears likely that a dilute solution of He³ in He⁴ will provide a uniquely suitable system for such an experiment.⁸

The remainder of this paper is devoted to a discussion of the extent to which a nuclear electric dipole moment can be made to interact with an externally applied electric field. From a classical point of view, it is apparent that if there is an electric field at the nucleus, the nuclear charge will cause the nucleus to accelerate, so that it will not be in a steady state. Thus, for a neutral atom that is at rest in or moves uniformly through a static electric field, the external field at the nucleus is exactly cancelled by the average field of the polarized cloud of electrons.¹⁰ As is shown in the next section, a quantum-mechanical treatment that includes the polarization of the electron cloud by the nuclear electric dipole as well as by the external field, again yields no first-order interaction between dipole and field.

A simple classical argument also shows that it is not helpful to use a time-dependent electric field. The equation of motion of the angular momentum vector \mathbf{J} of a classical electric dipole \mathbf{u} in an electric field \mathbf{E} is $d\mathbf{J}/dt = \mathbf{u} \times \mathbf{E}$. Now we can put $\mu = eD$, where D is probably less than 5×10^{-20} cm, so that for any reasonable value of E , the precession period will be very long. Thus we can assume that \mathbf{u} is nearly fixed in space, say along the z axis, and calculate the rotation of \mathbf{u} , and hence of the parallel vector \mathbf{J} , about the x and y axes. The angular velocity about the x axis is $d\theta_x/dt = -\mu E_x/J$, and there is a similar relation for the rotation about the y axis. The x component of the acceleration of the nucleus, of charge Ze and mass AM , is given by $dv_x/dt = ZeE_x/AM$. Thus $\Delta\theta_x = -\mu AM \Delta v_x / JZe = -(\gamma A / IZ)(\Delta v_x / c)$, where we have put $J = I\hbar$ and expressed D as a multiple γ of the nucleon Compton wavelength \hbar/Mc . The quantity γ should not exceed 10^{-7} if there is a charged vector boson that mediates the weak interactions with a substantial breakdown of time reversal invariance, and should be much smaller if the intermediate boson does

not exist.¹¹ Then since A/IZ is of order unity, and $\Delta v_x/c$ is much smaller than unity, we find that $\Delta\theta_x$ is unobservably small.¹²

We note in passing that a nucleus in a terrestrial laboratory is in fact subjected to an average electric field, although it is very small. The weight AMg of the nucleus must be counterbalanced by a vertical electric field AMg/Ze , where g is the local acceleration of gravity. However, the resulting precession period, $2\pi cIZ/\gamma Ag$, is far too long to be observed.

The quantum mechanics of a system of charged electric dipoles in an external electrostatic potential of arbitrary form is worked out in Sec. II. It is shown that if the over-all system is electrically neutral, there is no term in the interaction energy that is of first order in the electric dipole moments, regardless of the magnitude of the external potential. This is true even if the particles are of finite size, provided that the charge and dipole moment of each have the same spatial distribution. Relativistic effects are considered in Sec. III, and shown to be inappreciable. The interaction energy of order $\mu^2 E$ for a number of point charges, one of which carries an electric dipole, is calculated in Sec. IV. It is found to have a form and magnitude that make it impossibly difficult to observe. The interaction energy of order μE , of a system of charged electric dipoles for which the spatial distributions of charge and dipole moment are different, is calculated in Sec. V. The effect of a nuclear magnetic dipole is included in Sec. VI, and the results of Secs. V and VI are specialized to hydrogen and helium in Sec. VII.

II. QUANTUM MECHANICS OF A SYSTEM OF CHARGED ELECTRIC DIPOLES

Most of the following analysis was originally developed by using Ehrenfest's theorem to relate certain matrix elements in the absence of electric dipole moments and applying this relation when the same particles carry dipoles. However, Bloch and Yang subsequently pointed out to the writer that a charged electric dipole is equivalent to an infinitesimally displaced charge, and that the calculation could be performed with the help of displacement operators. The analysis is essentially the same for the two methods; but since a more compact and flexible development can be achieved by using displacement operators, only this procedure has been employed for the full calculation.

The nonrelativistic Hamiltonian for a system of particles of finite size, with mass m_i , charge e_i , electric dipole moment \mathbf{u}_i , and center-of-mass coordinate \mathbf{r}_i , in

⁸ B. S. Deaver, Jr. and W. M. Fairbank, Phys. Rev. Letters **7**, 43 (1961); R. Doll and M. N abauer, Phys. Rev. Letters **7**, 51 (1961).

¹⁰ R. L. Garwin and L. M. Lederman, Nuovo Cimento **11**, 776 (1959).

¹¹ T. D. Lee (private communication to W. M. Fairbank).

¹² For a discussion of a related effect, see P. A. Franken and H. S. Boyne, Phys. Rev. Letters **2**, 422, 3, 67 (1959).

an external electrostatic potential $\phi(\mathbf{r})$, may be written:

$$\begin{aligned}
 H &= T + V_0 + V + U + W, \\
 T &= -\sum_i (\hbar^2/2m_i)\nabla_i^2, \\
 V_0 &= \sum_{i>j} \sum e_i e_j \int \int \rho_{iC}(\mathbf{r}) \rho_{jC}(\mathbf{r}') \\
 &\quad \times |\mathbf{r}_i - \mathbf{r}_j + \mathbf{r} - \mathbf{r}'|^{-1} d^3r d^3r', \\
 V &= \sum_i e_i \int \rho_{iC}(\mathbf{r}) \phi(\mathbf{r}_i + \mathbf{r}) d^3r, \\
 U &= \sum_{i \neq j} \sum e_i \mathbf{u}_j \cdot \int \int (\mathbf{r}_i - \mathbf{r}_j + \mathbf{r} - \mathbf{r}') \rho_{iC}(\mathbf{r}) \rho_{jM}(\mathbf{r}') \\
 &\quad \times |\mathbf{r}_i - \mathbf{r}_j + \mathbf{r} - \mathbf{r}'|^{-3} d^3r d^3r', \\
 \text{and} \\
 W &= \sum_i \mathbf{u}_i \cdot \nabla_i \int \rho_{iM}(\mathbf{r}) \phi(\mathbf{r}_i + \mathbf{r}) d^3r.
 \end{aligned} \tag{1}$$

Direct dipole-dipole interaction terms, of order $\mu_i \mu_j$, have been neglected. The charge and dipole moment distribution functions, ρ_{iC} and ρ_{iM} , are normalized to unit volume integral. Each of the \mathbf{u}_i is assumed to be a quantum operator, the structure of which depends on the spin of that particle.

We define the infinitesimal displacement operator¹³

$$Q = \sum_i (\mathbf{u}_i \cdot \mathbf{p}_i) / (e_i \hbar), \tag{2}$$

where $\mathbf{p}_i = -i\hbar \nabla_i$ is the momentum operator for the i th particle. It is easily seen that Q commutes with T , and that

$$i[Q, V_0] = U', \quad i[Q, V] = W',$$

where U' and W' are the same as U and W except that ρ_{iM} is replaced by ρ_{iC} . Thus, if we call the Hamiltonian in the absence of dipole moments

$$H_0 = T + V_0 + V,$$

the full Hamiltonian (1) may be written:

$$\begin{aligned}
 H &= H_0 + i[Q, H_0] + \Delta U + \Delta W, \\
 \Delta U &= U - U', \quad \Delta W = W - W'.
 \end{aligned} \tag{3}$$

In the remainder of this section we shall assume that $\rho_{iC} = \rho_{iM}$, so that $\Delta U = \Delta W = 0$. Then Eqs. (2) and (3) show that H is the same as H_0 except for the displacement of each particle by the vector \mathbf{u}_i/e_i , provided that these vectors are regarded as being infinitesimal. This is in agreement with the classical view of a charged dipole. The infinitesimal displacement can also be written in terms of the finite displacement operator e^{iQ} by subtracting out the higher order terms¹³:

$$H = e^{iQ} H_0 e^{-iQ} + \frac{1}{2} [Q, [Q, H_0]] + \dots \tag{4}$$

¹³ Q is closely related to the helicity operators for the particles. Note that Q is Hermitian, and that e^{iQ} is unitary.

All terms except the first on the right side of Eq. (4) are of second or higher order in the μ_i , as are the neglected dipole-dipole terms. Thus to first order in the μ_i , the eigenfunctions u_n of H_0 , which satisfy the Schrödinger equation

$$H_0 u_n = E_n u_n, \tag{5}$$

determine in a simple way the eigenfunctions $e^{iQ} u_n$ of H , which satisfy the Schrödinger equation

$$H(e^{iQ} u_n) = E_n(e^{iQ} u_n) \tag{6}$$

with the same eigenvalues E_n . We implicitly assume here that stationary states u_n exist, which implies that the total charge $\sum_i e_i$ of the system is zero.

Now the energy eigenvalues of Eq. (5) do not depend on the μ_i , since no electric dipole moments appear in H_0 . We conclude that this is also true of the eigenvalues of Eq. (6), so that there is no interaction energy of first order in the dipole moments. This result depends on the assumption that the charge and moment distribution functions are the same, but is valid for an external potential of arbitrary form and magnitude.

III. RELATIVISTIC EFFECTS FOR A MASSIVE ELECTRIC DIPOLE

The discussion of Sec. II is based on the nonrelativistic Schrödinger equation. While a fully relativistic treatment of a system of interacting particles cannot be made, it is not difficult to consider the physically interesting case of an atom, in which the particle most likely to be carrying the electric dipole moment is the massive nucleus, and the electrons are Dirac particles. As in the preceding section, our procedure will consist in seeing to what extent the terms in the Hamiltonian that are proportional to μ are generated by displacement of the remaining terms. If the displaced terms agree with the μ -proportional terms, there is no first order interaction between μ and E . Since relativistic effects are expected to be small in atomic systems, we are justified in neglecting the finite size of the nucleus in this section.

We note first that for a hydrogen atom with neglect of nuclear motion, T in Eq. (1) is to be replaced by the Dirac operator for the electron: $\beta mc^2 + c\boldsymbol{\alpha} \cdot \mathbf{p}$. Also, since nuclear motion is neglected and the center of mass of the system may be assumed to be at rest, the momentum operator for the nucleus is conveniently replaced by $-\mathbf{p}$, so that $Q = -\mathbf{u} \cdot \mathbf{p} / e\hbar$. It is then easily seen along the lines of Sec. II that there is no first-order interaction. This argument can be generalized to an atom with Z electrons, in which case $Q = -(\mathbf{u} Z e \hbar) \cdot \sum_i \mathbf{p}_i$. The interactions of the electrons with each other do not alter the result, since they depend only on the relative coordinates and momenta of the electrons, and are unaffected by the total momentum operator that appears in Q . This argument breaks down if there are two or more neighboring nuclei which possess elec-

tric dipole moments, although the effect of the breakdown is probably very small. However, this is not the experimental situation envisaged in Sec. I (dilute solution of He^3 in He^4), so we may restrict our considerations to a single atom.

Nuclear motion can be taken into account by using the Breit equation¹⁴; it is sufficient for our present purpose to consider only two particles, the nucleus and electron of a hydrogen atom. An examination of Eqs. (39.14) and (42.1) of Ref. 14 when an external electric field but no magnetic field is present, shows that the several terms are affected differently by the addition of an electric dipole moment to one of the particles and by the displacement operation. They are also of various orders of magnitude. As will be shown in Sec. VII, we are justified in neglecting terms that are of order v_N^2/c^2 but not terms of order $v_N v_e/c^2$, with respect to the energy of the atom, where v_N and v_e are the nuclear and electron velocities, respectively.

The term H_2 of Ref. 14, which represents the retardation of the interaction between the charges of nucleus and electron, is of relative order $v_N v_e/c^2$. However, the term generated by displacement of H_2 is the same as the modification produced by replacing the nuclear charge by a charged electric dipole; thus H_2 has no first-order effect. The term H_5 , which represents the interaction between the magnetic dipole moments of nucleus and electron, contributes nothing since the electron is assumed to be unpolarized, so that the expectation value of the electron spin is zero. The only term that might make a contribution of interesting magnitude is that part of H_3 which arises from the interaction between the nuclear magnetic moment and the current caused by the motion of the electron charge. This is not really a relativistic effect, and will be considered in Sec. VI.

Higher order relativistic effects of the nuclear motion, not included in the Breit equation, are too small to be of interest. Similar terms for the electron are also too small if multiplied by v_N/c , and otherwise are included in the discussion of the second paragraph of this section. Another relativistic effect not included in the Breit equation is the Foldy interaction¹⁵ between the anomalous magnetic moment of the nucleus and the charge of the electron; this is of relative order v_N^2/c^2 , and hence negligible.

IV. SECOND-ORDER INTERACTION FOR A POINT ELECTRIC DIPOLE

The interaction energy of second order in the dipole moments may be obtained from the double commutator term of Eq. (4). For simplicity, we restrict ourselves in this section to a single point dipole of mass m_0 , charge e_0 , electric dipole moment $\mu\sigma$, and coordinate \mathbf{r} , and a

¹⁴ H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic Press Inc., New York, 1957), pp. 181, 193.

¹⁵ L. L. Foldy, Phys. Rev. **87**, 688 (1952).

number of point charges described by m_i , e_i , and \mathbf{r}_i . We choose a dipole of spin $\frac{1}{2}$ for definiteness, so that the components of σ are the Pauli spin matrices. Since any second-order interaction is expected to be very small, we are justified in neglecting the finite sizes of the particles.

A straightforward calculation gives:

$$\frac{1}{2}[Q, [Q, V_0]] = (2\pi\mu^2/e_0) \sum_i e_i \delta(\mathbf{r} - \mathbf{r}_i) + (\mu^2/e_0\hbar) \sigma \cdot \left\{ \sum [e_i(\mathbf{r} - \mathbf{r}_i)/|\mathbf{r} - \mathbf{r}_i|^3] \times \mathbf{p} \right\}, \quad (7)$$

$$\frac{1}{2}[Q, [Q, V]] = (\mu^2/e_0\hbar) \sigma \cdot (\mathbf{E} \times \mathbf{p}), \quad (8)$$

where $\mathbf{E} = -\nabla\phi$ is the external electric field at the point \mathbf{r} . The first term on the right side of (7) does not involve σ , and hence does not lead to a precession; in a system of linear dimensions a_0 , it produces an energy level shift of order μ^2/a_0^3 , which at best is unobservably small in the atomic case. The second term on the right side of (7) leads in general to a spin-field interaction if its expectation value is calculated with an eigenfunction of H_0 that includes the effect of the external field interaction V . However, if the zero-field eigenfunction of $T+V_0$ is spherically symmetric, V will distort it along the direction of the external field, and the expectation value of the vector product will be zero. Similarly, the right side of (8), which already contains both σ and \mathbf{E} , has a vanishing expectation value for a spherically symmetric state.

Even if the zero-field eigenfunction is not assumed to be spherically symmetric, the order of magnitude of the spin-field interaction cannot exceed $\mu^2 E/ea_0$, which at best is much smaller than that obtained in Sec. VII.

V. FIRST-ORDER INTERACTION FOR ELECTRIC DIPOLES OF FINITE SIZE

We now return to Eqs. (3) and (4), and write H to first order in the μ_i :

$$H = e^{iQ} H_0 e^{-iQ} + \Delta U + \Delta W.$$

We no longer assume that $\rho_{iC} = \rho_{iM}$, but define the difference distribution function

$$\rho_i(\mathbf{r}) = \rho_{iC}(\mathbf{r}) - \rho_{iM}(\mathbf{r}). \quad (9)$$

Since ρ_{iC} and ρ_{iM} are normalized, the volume integral of ρ_i is zero. It is sufficient for the experimental situation to regard the electric field as uniform, in which case $\Delta W = 0$. Further, the particles may be assumed to be small in comparison with their mean separations, so that we need calculate only the leading term in a power series of the ratio of size to separation. Thus in the expression for ΔU , which contains $\rho_{iC}(\mathbf{r})\rho_j(\mathbf{r}')$ in its integrand, we can replace ρ_{iC} by a δ function:

$$\Delta U = - \sum_{i \neq j} e_i \mathbf{p}_j \cdot \int (\mathbf{r}_{ij} - \mathbf{r}) \rho_j(\mathbf{r}) \times |\mathbf{r}_{ij} - \mathbf{r}|^{-3} d^3r, \quad \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j. \quad (10)$$

In calculating the expectation value of ΔU , we may use the eigenfunction u_n of Eq. (5) rather than the eigenfunction $e^{iQ}u_n$ of Eq. (6), since we are only interested in the interaction of first order in the μ_i . We may also use the u_n that are calculated for point charges, since we are only interested in the leading term for small particle sizes. We must of course include the V term in H_0 in calculating u_n , but need do so only to first order. It is apparent, then, that u_n is a product of space and spin functions, so that the latter can be used to calculate the expectation value of \mathbf{u}_j , which we call $\langle \mathbf{u}_j \rangle$. Now we expect that $\langle \mathbf{u}_j \rangle$ is a symmetry axis for $\rho_j(\mathbf{r})$, so that we can write

$$\rho_j(\mathbf{r}) = \sum_l f_{jl}(r) P_l(\cos\chi), \quad (11)$$

where χ is the angle between \mathbf{r} and $\langle \mathbf{u}_j \rangle$.

The integral on the right side of Eq. (10) can now be evaluated, and expressed in terms of the angle θ between \mathbf{r}_{ij} and $\langle \mathbf{u}_i \rangle$. The result for the spin-expectation value of ΔU is

$$\begin{aligned} \langle \Delta U \rangle = & - \sum_{i \neq j} \sum_l e_i \langle \mu_j \rangle \sum_l (4\pi/2l+1) \\ & \times \left[(l+1) P_{l+1}(\cos\theta) \int_0^{r_{ij}} (r/r_{ij})^{l+2} f_{jl}(r) dr \right. \\ & \left. - l P_{l-1}(\cos\theta) \int_{r_{ij}}^{\infty} (r_{ij}/r)^{l-1} f_{jl}(r) dr \right], \quad (12) \end{aligned}$$

where $\langle \mu_j \rangle$ is the magnitude of the vector $\langle \mathbf{u}_j \rangle$. If the zero-field eigenfunction of $T+V_0$ is spherically symmetric, V will introduce a P_1 -type dependence on the angle between \mathbf{r}_{ij} and \mathbf{E} , so that only the $P_1(\cos\theta)$ terms in Eq. (12) will contribute. Thus only f_{j0} and f_{j2} appear in the spin-field interaction.¹⁶ Numerical results based on Eq. (12) will be presented in Sec. VII.

VI. FIRST-ORDER INTERACTION FOR A COMBINED ELECTRIC AND MAGNETIC DIPOLE

It was remarked near the end of Sec. III that the interaction between the nuclear magnetic moment and the current caused by the motion of the electron charge might contribute an effect of interesting magnitude. We need only consider the case of a single particle (nucleus) of mass m_0 , charge e_0 , electric dipole moment $\mu\sigma$, magnetic dipole moment $(\kappa e\hbar/2Mc)\sigma$, and coordinate \mathbf{r} , and a number of point charges described by m_i , e_i , and \mathbf{r}_i ; M is the proton mass, and $e\hbar/2Mc$ is the nuclear magneton. Since we are especially interested in He^3 , the components of σ are the Pauli spin matrices. The Hamiltonian of Eq. (1) must then be replaced by

$$\begin{aligned} H = & T + V_0 + V + U + W + H_M, \\ H_M = & -(\kappa e\hbar/2Mc)\sigma \\ & \cdot \sum_i (e_i/m_i c) [(\mathbf{r}_i - \mathbf{r}) \times \mathbf{p}_i] |\mathbf{r}_i - \mathbf{r}|^{-3}. \quad (13) \end{aligned}$$

¹⁶ In this case there will also be a field-independent energy that involves f_{j1} ; this does not appear to be of experimental interest.

Since the effects of H_M are expected to be small, we are justified in neglecting the finite size of the nucleus. The expression (13) for H_M comes from the Breit equation,¹⁴ and can also be derived classically.

There are now two equivalent ways in which we can proceed. First, we can define H_0 as $T+V_0+V+H_M$, in which case we have, to first order in μ ,

$$H = e^{iQ} H_0 e^{-iQ} - i[Q, H_M].$$

Since the commutator contains μ but not E , we must then calculate its expectation value for the appropriate eigenfunction u_n of H_0 that includes the effect of V ; however, since we only wish to go to first order in H_M , we can neglect its effect on this eigenfunction. Second, we can as an alternative define H_0 as in Sec. II, in which case we have, again to first order in μ ,

$$H = e^{iQ} H_0 e^{-iQ} + H_M.$$

The perturbing term now contains neither μ nor E , so that we must calculate the expectation value of H_M for $e^{iQ}u_n$ rather than for u_n . To first order in μ , this gives the result just quoted.

As in Sec. V, we can write u_n as a product of space and spin functions and use the latter to calculate the spin expectation value of $-i[Q, H_M]$. The result is

$$\begin{aligned} & -i\langle [Q, H_M] \rangle \\ & = -(\mu\kappa e/2e_0 M c) \langle \sigma_z \rangle \sum_i (e_i/m_i c) \\ & \quad \times \{ [\mathbf{p}_x, \{ (z_i - z) p_{ix} - (x_i - x) p_{iz} \} | \mathbf{r}_i - \mathbf{r} |^{-3}]_+ \\ & \quad + [\mathbf{p}_y, \{ (z_i - z) p_{iy} - (y_i - y) p_{iz} \} | \mathbf{r}_i - \mathbf{r} |^{-3}]_+ \\ & \quad + \dots + \dots \}, \quad (14) \end{aligned}$$

where the square brackets are anticommutators, and the two additional terms are derived from the first by cyclic permutation of the subscripts. Numerical results based on Eq. (14) will be presented in the next section.

VII. RESULTS FOR HYDROGEN AND HELIUM

Finite Size Effect

The expectation value of $\langle \Delta U \rangle$ given by Eq. (12) is easily calculated for a hydrogen atom in its ground state. In Eq. (12), we put $\langle \mu_j \rangle = \langle \mu \rangle$ for the nucleus, $e_i = -e$ for the electron, and $\mathbf{r}_{ij} = \mathbf{r}_1$ for the vector from nucleus to electron. The normalized wave function, correct to first order in V , has been given by Kotani¹⁷:

$$u_0 = (\pi a_0^3)^{-1/2} e^{-r_1/a_0} [1 - (\mathbf{r}_1 \cdot \mathbf{E}/2e)(r_1 + 2a_0)], \quad a_0 = \hbar^2/me^2. \quad (15)$$

As remarked at the end of Sec. V, the only part of

¹⁷ M. Kotani, *Quantum Mechanics* (Yuwani Book Co., Tokyo, 1951), Vol. I, p. 127.

Eq. (12) that contributes to the spin-field interaction is

$$4\pi e\langle\boldsymbol{\mu}\rangle\cos\theta\left[\int_0^{r_1}(r/r_1)^2f_0(r)dr-\int_r^\infty(2r_1/5r)f_2(r)dr\right]. \quad (16)$$

The expectation value of (16) with the wave function (15) is readily calculated to be

$$(16\pi/3a_0^2)\langle\langle\mathbf{u}\rangle\cdot\mathbf{E}\rangle\int_0^\infty r^4[f_0(r)+(4/25)f_2(r)]dr. \quad (17)$$

We have made use of the fact that $\int_0^\infty r^2f_0(r)dr=0$ since the volume integral of (11) is zero, and have also assumed that the spatial extent of f_0 and f_2 is much smaller than a_0 . The expression (17) may be written in terms of moments of the difference distribution function $\rho(\mathbf{r})$ given by Eqs. (9) and (11):

$$R_i^2=\int r^2P_i(\cos\chi)\rho(\mathbf{r})d^3r=(4\pi/2l+1)\int_0^\infty r^4f_i(r)dr.$$

Thus the spin-field interaction energy is

$$[4\langle\langle\mathbf{u}\rangle\cdot\mathbf{E}\rangle/3a_0^2][R_0^2+\frac{4}{3}R_2^2]. \quad (18)$$

The corresponding calculation for a helium atom in its ground state cannot of course be carried through without extensive numerical work. However, a crude estimate, which is sufficient for the present purpose, can be made by using a wave function similar to (15):

$$(Z^3/\pi a_0^3)e^{-Z(r_1+r_2)/a_0}[1-(\mathbf{r}_1\cdot\mathbf{E}/4Ze)(Zr_1+2a_0)-(\mathbf{r}_2\cdot\mathbf{E}/4Ze)(Zr_2+2a_0)]. \quad (19)$$

This is so arranged that the total electric field at the nucleus is zero, as is the case with (15). The value of the effective charge parameter Z that minimizes the ground-state energy of the atom is $27/16$. The spin-field interaction energy then turns out to be Z^2 times the hydrogen value (18).

Recent experiments of Collard *et al.*¹⁸ in which high energy electrons are scattered from He^3 , give 1.97×10^{-13} cm for the root-mean-square (rms) radius of the charge density of the He^3 nucleus, and 1.69×10^{-13} cm for the rms radius of the magnetic moment density. If it is assumed that whatever electric dipole moment is present is distributed in the same way as the magnetic dipole moment, then

$$R_0^2=[(1.97)^2-(1.69)^2]\times 10^{-26}\text{ cm}^2=1.02\times 10^{-26}\text{ cm}^2.$$

Nothing is known of R_2 , and it may safely be presumed to be negligibly small. We thus expect the spin-field interaction energy in He^3 that arises from the finite size of the nucleus to be roughly equal to $1.4\times 10^{-9}\times\langle\langle\mathbf{u}\rangle\cdot\mathbf{E}\rangle$.

¹⁸ H. Collard, R. Hofstadter, A. Johansson, R. Parks, M. Ryneveld, A. Walker, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. Letters **11**, 132 (1963).

Magnetic Moment Effect

In a light atom, $v_e/c\sim e^2/\hbar c=1/137$, and $v_N/c\sim(m/M)(v_e/c)$, where m and M are the electron and proton masses, respectively. Thus the parameter v_Nv_e/c^2 that appeared in Sec. III is of order 10^{-8} , and so considerably larger than the multiplying factor just obtained on the basis of the finite size of the nucleus. The magnetic moment effect is of this order of magnitude, and so must be calculated. On the other hand, terms of order $v_N^2/c^2\sim 10^{-11}$ can be neglected.

In calculating the expectation value of $-i\langle[Q,H_M]\rangle$ given by Eq. (14), it is convenient to work in the center-of-mass coordinate system, in which case $\mathbf{p}=-\sum_i\mathbf{p}_i$. We note also that for the wave functions (15) and (19), only the $\langle\sigma_z\rangle$ part of (14) contributes. For hydrogen, we put $\mu\langle\sigma_z\rangle=\langle\mu_z\rangle$ and $e_0=e$ for the nucleus, $e_i=-e$ for the electron, and $\mathbf{r}_i-\mathbf{r}=\mathbf{r}_1$ for the vector from nucleus to electron. The calculation is most readily performed by doing a partial integration on each term, after which the two parts of each anticommutator are seen to be the same. The result for the wave function (15) is¹⁹

$$(5\kappa/3)(m/M)(e^2/\hbar c)^2\langle\langle\mathbf{u}\rangle\cdot\mathbf{E}\rangle. \quad (20)$$

For helium, we use the approximate wave function (19) with $Z=27/16$. In (14), e_0 must now be replaced by $2e$, and $\mathbf{p}=-\mathbf{p}_1-\mathbf{p}_2$; there are, however, no cross terms between the $i=1$ and $i=2$ parts of the expectation value of (14). The interaction then turns out to be $\frac{1}{2}Z^2$ times the hydrogen value (20). With $\kappa=-2.127$ for He^3 , the spin-field interaction energy is roughly equal to $-1.5\times 10^{-7}\langle\langle\mathbf{u}\rangle\cdot\mathbf{E}\rangle$, or about a hundred times larger than the finite size effect.

Discussion

From a classical point of view, there can be no average electric field at the nucleus unless some non-electric force is available to keep the nucleus from accelerating under the influence of this electric field. In the finite size effect, this force is supplied by the non-electric interactions between nucleons and mesons. These give the nucleus a finite size, and make it possible for whatever electric dipole moment it may possess to be in a region where the electric field is not exactly zero.

In the magnetic moment effect, the needed force arises in the following way. The external electric field distorts the electron distribution from its normal spherical shape, and makes it possible for the electron current to produce a magnetic field at the nucleus, the gradient of which can act on the nuclear magnetic moment to supply the nonelectric force. However, the

¹⁹ This applies to light hydrogen and tritium (nuclear spin $I=\frac{1}{2}$); in general, Eq. (20) must be divided by $2I$. For deuterium, the electric quadrupole effect is smaller than the magnetic moment effect by a factor of order m/M , and hence can be neglected.

expectation value of this magnetic field gradient is zero, and indeed the calculation that leads from (13) to (14) would give nothing if the electric and magnetic dipole operators were classical vectors, or even if they were not classical but their components commuted with each other. It is the fact that each dipole moment is proportional to σ , the components of which do not commute, that leads to a nonvanishing interaction.²⁰

If the parameter γ of Sec. I is 10^{-7} and E is 10^5 V/cm, the precession rate of He^3 nuclei caused by the magnetic moment effect is roughly half a degree per day. It

²⁰ The same noncommutativity effect was found in connection with a PT -noninvariant version of electrodynamics by M. Sachs and S. L. Schwebel, *Ann. Phys. (N. Y.)* **8**, 475 (1959).

seems possible that considerably smaller precession rates can be measured.⁸

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Elastic Scattering of 17-MeV Protons by Heavy Nuclei*

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Absolute differential cross sections for elastic scattering of protons from Ta^{181} , W , Pb^{208} , and Bi^{209} have been measured at a center-of-mass energy of 17.00 ± 0.05 MeV at angular intervals of five degrees ranging from 20 to 170° . The estimated relative standard deviation of each point is 3% while the absolute cross-section scale is uncertain by 5% for Pb and Bi and by 10% for Ta and W . The scattered protons were detected by a $\text{NaI}(\text{Tl})$ scintillation counter with an over-all energy resolution of 2.5%. Consequently, while all inelastically scattered protons are rejected for Bi and Pb , a small contribution of inelastic protons from the lowest levels in Ta and the tungsten isotopes is included in the measured cross sections. For Ta and W the diffraction pattern appears damped at backward angles relative to the heavier two targets to a greater extent than may be attributed to the effects of an inelastic scattering component.

I. INTRODUCTION

ELASTIC differential cross sections for the scattering of particles by nuclei can be measured with relative precision and have had an historic role as a formidable test of nuclear models. The present work was undertaken to aid in determining the role of a deformation parameter in the nuclear optical model. In 1955, Hahn and Hofstadter¹ found that the scattering of 183-MeV electrons by Ta , W , and U gave rise to diffraction patterns with large angle oscillations less pronounced than in Pb^{208} , Au , and Bi . In a companion paper, Downs *et al.*² showed that a nuclear form factor including a quadrupole charge distribution could better reproduce the observations. Margolis³ summarized the situation in 1959 with particular reference to the success of the work of Chase, Wilets, and Edmonds⁴

who reproduced some of the detailed structure of the neutron strength function using a spheroidal optical potential. Schey⁵ has shown that modification of the Bjorklund-Fernbach type of optical potential improves the fit to the data of Beyster *et al.*⁶ for the scattering of 7-MeV neutrons by Ta . The deformation giving the best fit was in reasonable agreement with Coulomb excitation measurements.⁷ Buck⁸ has extended the optical model generalization to the simultaneous prediction of proton elastic and inelastic scattering. A more recent experiment of Hudson *et al.*⁹ on the scattering of 15.2-MeV neutrons by Ta , Th , and U shows once again the characteristic flattening of the diffraction structure at large angles relative to Bi and relative to predictions with a spherical optical model. Data on the four nuclei in this report supplemented by comparable earlier 17-MeV

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⁷ See, for example, K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, *Rev. Mod. Phys.* **28**, 432 (1956).

⁸ B. Buck, *Phys. Rev.* **130**, 712 (1963).

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